# Performance evaluation for a diversity decoding scheme under multipath environment with OFDM


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**Abstract**

This paper is concerned with the performance evaluation for the diversity decoding scheme in which space diversity technique and iterative decoding for non systematic convolutional error-correcting code are effectively combined. The concept of the diversity decoding is applied to an OFDM system (e.g. the digital terrestrial TV broadcasting system). This paper shows the performance evaluation for the diversity decoding scheme under multipath environment with OFDM.


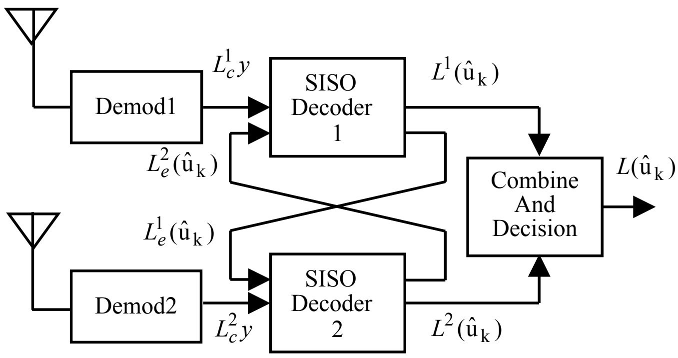


Fig.1. Diversity decoder

## 1. Introduction

It is well known that OFDM (Orthogonal Frequency Division Multiplexing) transmission systems such as the digital terrestrial TV broadcasting system and MMAC (Multimedia Mobile Access System) have been developed to overcome the effect of multipath fading.

In the mobile communications, however, it is necessary to improve the performance moreover because of severe propagation environment. For this purpose, the diversity decoding scheme, in which space diversity technique and error correcting decoding are effectively combined, has been developed [1].

It is considered important to evaluate the performance of the diversity decoding scheme under multipath environment. In this paper, therefore, the performance of the diversity decoding scheme applied to an OFDM system under multipath environment is evaluated through computer simulations.

## 2. Diversity decoding scheme

Figure 1 shows the block diagram of the diversity decoding scheme. In this figure, the information gained by the decoding process on the first diversity branch is utilized for decoding on the second diversity branch and vice versa. That is, each of the received signals from two antennas is applied to respective SISO (Soft-in Soft-out) decoder. SISO decoder and iterative decoding are described as follows.

### 2.1. SISO decoder

SISO decoder decodes by soft decision in log-likelihood domain, and the soft-output LLR (Log-Likelihood Ratio) $L(\hat{u}_k)$ can be expressed as:

$$L(\hat{u}_k) = L(u_k) + L_c \cdot y + L_e(\hat{u}_k), \tag{1}$$

where $L(u_k)$ is *a priori* LLR for $k$-th information bit, and it is LLR of the probability that the transmitted bit is 1 or -1.

$$L(u_k) = \log \frac{P(u_k = +1)}{P(u_k = -1)} \tag{2}$$

$L_c \cdot y$ is the channel state information for each encoded bit. $L_c \cdot y$ can be expressed by the following processes.

The signal $y_k$ received through fading and the AWGN channels is expressed as :

$$y_k^m = a_k^m x_k + n_k^m \quad , \qquad (3)$$

where $x_k$ is $k$-th transmitted signal, $m$ represents diversity branches, $a$ is a random amplitude value caused by fading, $n$ is Gaussian random variable.

When the transmitted signal is $x$ and the received signal is $y$, *a posteriori* probability is:

$$P(x \mid y) = \frac{p(y \mid x)p(x)}{p(r)} \quad , \qquad (4)$$

where $P(y \mid x)$ is determined by the probability density function of AWGN with variance $\sigma^2$ expressed as follows:

$$P(y \mid x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-ax)^2}{2\sigma^2}\right) \quad , \qquad (5)$$

Log-likelihood ratio $L(x \mid y)$ for *a posteriori* probability $p(x \mid y)$ is calculated as follows:

$$\begin{aligned}
L(x \mid y) &= \ln\left[\frac{P(x=+1 \mid y)}{P(x=-1 \mid y)}\right] \\
&= \ln\left[\frac{P(y \mid x=+1)P(s=+1)}{P(y \mid x=-1)P(s=-1)}\right] \\
&= \ln\left[\frac{P(y \mid x=+1)}{P(y \mid x=-1)}\right] + \ln\left[\frac{P(s=+1)}{P(s=-1)}\right] \\
&= \ln\left[\frac{\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(y-a)^2}{2\sigma^2}\right)}{\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(y+a)^2}{2\sigma^2}\right)}\right] + L(s) \quad , \qquad (6) \\
&= \frac{2a}{\sigma^2} y + L(s) \\
&= 4a\left(\frac{E_s}{N_0}\right) y + L(s) \\
&= Lcy + L(s)
\end{aligned}$$

where $L(s)$ is *a priori* information (*a priori* LLR), $E_s / N_0$ is the ratio of symbol energy to noise power spectral density for unfaded condition. Then, $L_c \cdot y$ is expressed as follows:

$$L_c y = 4\left(\frac{E_s}{N_0}\right) m_k \cdot y \qquad (7)$$

$L_e(\hat{u}_k)$ is the extrinsic information that is the metric gained through the decoding process with respect to a decoded path. The extrinsic information can be obtained by SOVA (Soft output Viterbi algorithm) in which soft output is obtained from soft input based on Viterbi algorithm [2]. At time $k$ and $i$-th path metric in SOVA is expressed as follows:

$$M_k\left(s^i\right) = M_{k-1}\left(s'^i\right) + \frac{1}{2}L(u_k)u_k^i + \frac{1}{2}\sum_{v=1}^{n} L_c y_{k,v} x_{k,v}^{(i)} \; , \qquad (8)$$

where $s^{(i)}$ denotes the state of the path $i$ at time k, $u_k^i$ is the information bit, and $x_{k,v}^{(i)}$ are the coded bits of path $i$ at time $k$. Using this metric, we calculate path metrics and get the path with the maximum metric through the Viterbi algorithm. The difference of the metric between selected path in time $k$ and discarded the other path in time $k$ is calculated as follows:

$$\triangle_k = M_k(s^i) - M_k(s^{i'}) \geq 0 \qquad (9)$$

This calculation is made for all times, and the minimum value of the differences is taken. Then, $L(\hat{u}_k)$ is expressed as:

$$L(\hat{u}_k) \approx u_k \cdot \min_{k=0,\cdots,\delta} \triangle_k \qquad (10)$$

where $\delta$ is the final time in trellis. Thus, the extrinsic information $L_e(\hat{u}_k)$ is obtained as follows:

$$L_e(\hat{u}_k) = L(\hat{u}_k) - L(u_k) \qquad (11)$$

### 2.2. Iterative decoding

The iterative decoding with the SISO decoder is done according to the following procedures. At the first decoding step, the metric obtained in each branch is given as follows:

$$\begin{aligned}
L^{(1,0)}(\hat{u}_k) &= L_c^1 y + L_e^{(1,0)}(\hat{u}_k) \\
L^{(2,0)}(\hat{u}_k) &= L_c^2 y + L_e^{(2,0)}(\hat{u}_k) \, ,
\end{aligned} \qquad (12)$$

where (1, 0) denotes diversity branch 1 and 0$^{\text{th}}$ iterative decoding step, respectively.

At the second decoding step (the first iterative decoding step), the extrinsic information $L_e(\hat{u}_k)$ is used as the new *a priori* LLR for another diversity branch. Thus, $L(\hat{u}_k)$ of each branch is expressed as:

$$\begin{aligned}
L^{(1,1)}(\hat{u}_k) &= L_e^{(2,0)}(\hat{u}_k) + L_c^1 y + L_e^{(1,1)}(\hat{u}_k) \\
L^{(2,1)}(\hat{u}_k) &= L_e^{(1,0)}(\hat{u}_k) + L_c^2 y + L_e^{(2,1)}(\hat{u}_k)
\end{aligned} \qquad (13)$$

After the third step, the calculation is repeated. At the ($N$+1)-th decoding step (the $N$-th iterative decoding step), $L(\hat{u}_k)$ of each branch is expressed as:

$$L^{(1,N)}(\hat{u}_k) = L_e^{(2,N-1)}(\hat{u}_k) + L_c^1 y + L_e^{(1,N)}(\hat{u}_k)$$
$$L^{(2,N)}(\hat{u}_k) = L_e^{(1,N-1)}(\hat{u}_k) + L_c^2 y + L_e^{(2,N)}(\hat{u}_k) \qquad (14)$$

At the final stage, the soft output is obtained from adding both of the last two extrinsic LLRs, becoming *a priori* LLRs, together with a pair of channel information, as follows:

$$L(\hat{u}_k) = L_c y_k^{(1)} + L_c y_k^{(2)} + L^{(1)}(u_k) + L^{(2)}(u_k) \qquad (15)$$

and based on these soft output values, the path is decided through Viterbi algorithm.

## 3. Performance evaluation

### 3.1. Simulation process

The performance of the diversity decoding scheme under the multipath environment is evaluated through simulation. As the multipath, the two-path model is used for simulations, in which parameters (the delay difference and D/U ratio) are changed.

The simulation is conducted based on the OFDM segment parameter (mode-3) for the digital terrestrial TV broadcasting system in Japan, which is shown in Table 1.

Table 1. OFDM segment parameter

| | |
|---|---|
| modulation | 64QAM OFDM |
| convolutional code | coding rate : 1/2<br>constraint length : 7 |
| guard interval duration | 31.5 μs（1/32） |
| FFT size | 8192 |
| number of sub- carriers | 5617 |

For setting the delayed wave, delay time is 0.1 – 30[μs], the phase difference between two branches is arbitrarily set between 0－180°, and D/U is assumed to be 3[dB].

The simulation process is summarized in the flow diagram shown in Fig.2. The following items are evaluated.

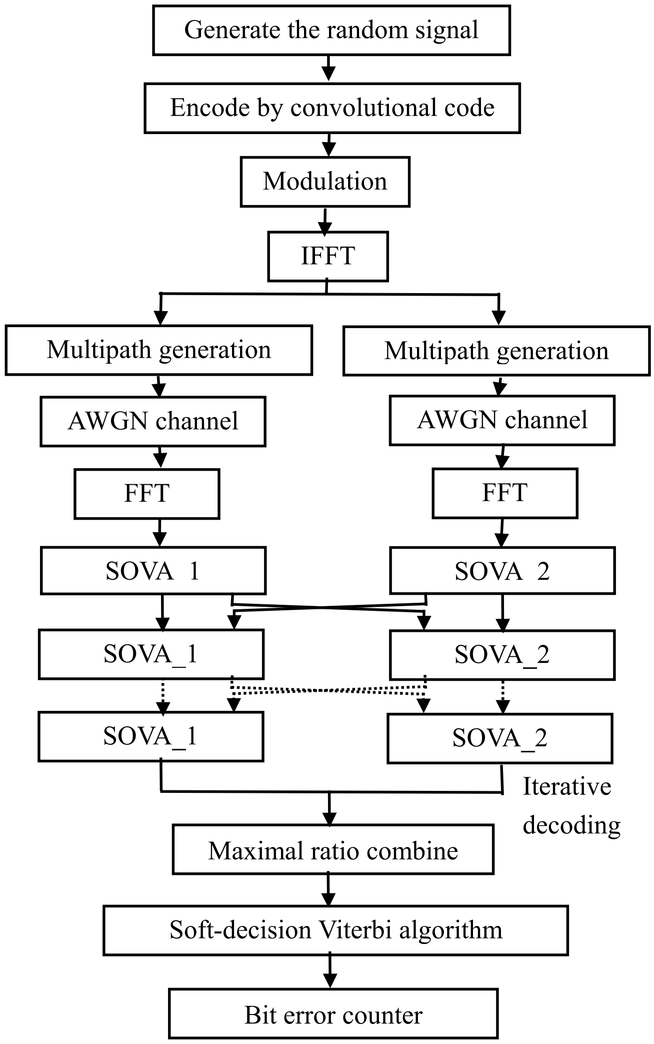


Fig.2. Flow diagram describing simulation process

1. Performance when phase difference on the delay wave between two branches is changed.
2. Performance when delay time of delay wave is changed.

For the setting of the delay wave in evaluation item 1, delay time and D/U ratio fixed by 1 [μs] and 3[dB]. For the setting of the delay wave in evaluation item 2, D/U ratio and phase difference fixed by 3[dB] and 180°. Moreover, number of iterative decoding was set to six times.

### 3.2. Simulated results

Figure 3 shows one example of the variation of amplitude versus frequency of the received signal when the phase difference of the delayed wave between two branches is assumed to be 180°. Delay time and D/U are 1[μs], and 3[dB], respectively. Figure 4 shows the bit-error-rate versus C/N (Carrier to Noise ratio) as a parameter of the phase difference of the delayed wave between two branches. Figure 5 shows the bit-error-rate as a parameter of delay time.

From the result shown in Fig.4, the effectiveness of the diversity decoding scheme is found to be remarkable when the phase difference of the delayed wave between two branches is 180°. It is also found that the diversity gain of about 13dB is obtained as compared with the performance without diversity at the error rate of $10^{-3}$.

From the result shown in Fig.5, the bit-error-rate at the delay time of 30μs is deteriorated because of the influence of the delayed wave, but a remarkable effect could be achieved as compared with the performance without diversity.

## 4. Conclusion

This paper discusses the performance evaluation for the diversity decoding scheme in which space diversity and error correcting decoding are effectively combined. Simulation results of the bit-error-rate performance of this scheme under multipath environment have been presented. As a result, it has been found that the diversity decoding scheme works effectively under the multipath environment.

## 5. Acknowledgement

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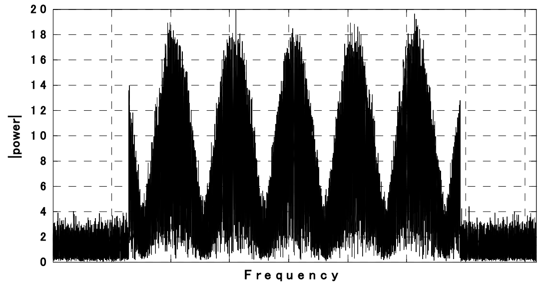
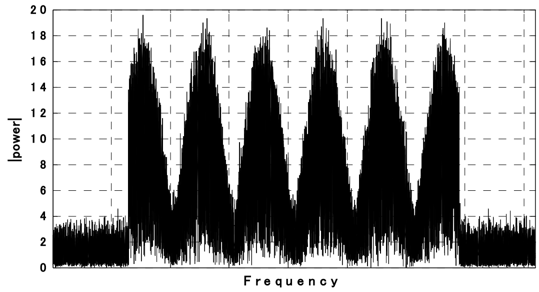
## References

[1] M.Ito, A.Ogawa and T.Yamazato : "Performance evolution of a diversity decoding scheme for punctured convolutional code", International Symposium on Information Theory and Its Applications (ISITA2002), Xi'an, China (2002-10)

[2] J. Hagenauer, E. Offer and L. Papke, "Iterative decoding of binary block and convolutional codes," IEEE Trans.,IT, vol.42, no.2, pp429-445, Mar.,1996




(a) Diversity branch 1




(b) Diversity branch 2

Fig.3 Variation of the amplitude vs. frequency (the phase difference ; 180°)

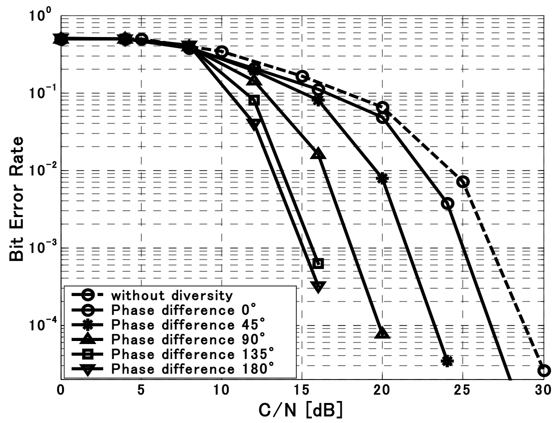



Fig.4 Bit-error-rates vs. C/N as a parameter of phase difference

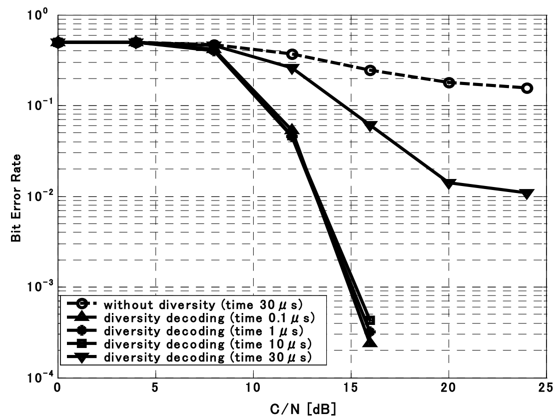



Fig.5 Bit-error-rates vs. C/N as a parameter of the delay time

# Performance evaluation for a diversity decoding scheme under multipath environment with OFDM

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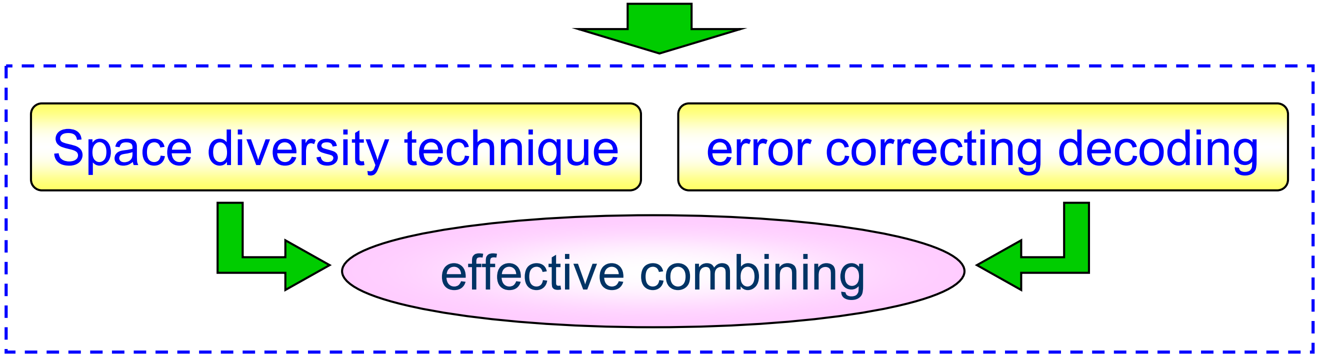
# Background

- It is well known that many OFDM systems have been developed to overcome the effect of multipath fading.
- In the mobile communications, it would be necessary to improve the performance more because of severe propagation environment.

# Objectives

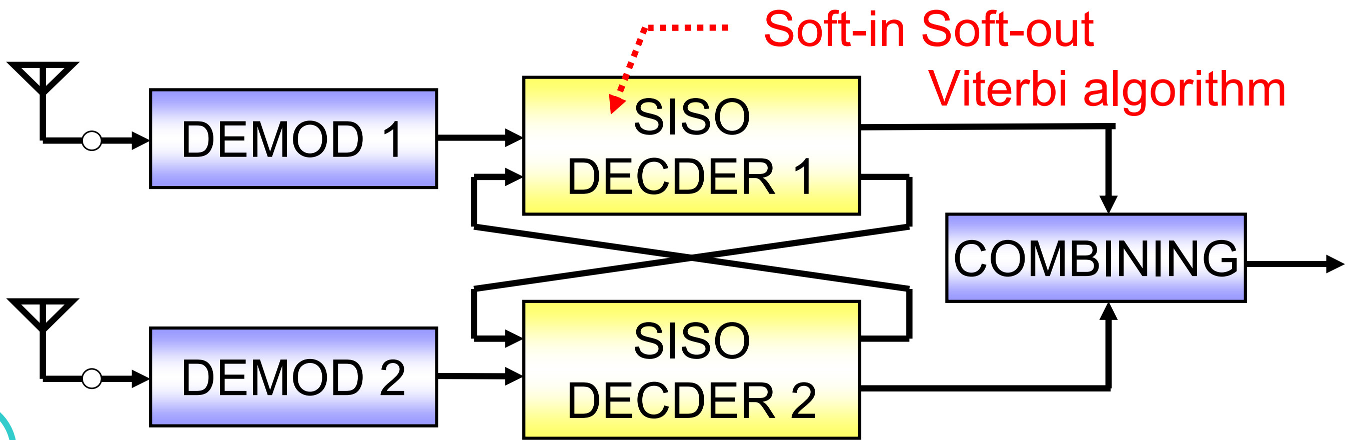
- For this purpose, we have developed the scheme called <u>the diversity decoding</u>.




- We have conducted a simulation to evaluate the performance of an OFDM system with this diversity decoding scheme under the multipath environment.

# Configuration of the diversity decoding

- Two diversity branches
- Iteration of error correcting decoding
- Diversity combining is made at the output of SISO (Soft-In Soft-Out) decoders



# Log-Likelihood Ratio(LLR)

- The Log-Likelihood Ratio(LLR) at the output from a SISO Decoder is expressed as

$$L(\hat{\mathrm{u}}_{\mathrm{k}}) = L(\mathrm{u}_{\mathrm{k}}) + L_c \cdot y + L_e(\hat{\mathrm{u}}_{\mathrm{k}})$$

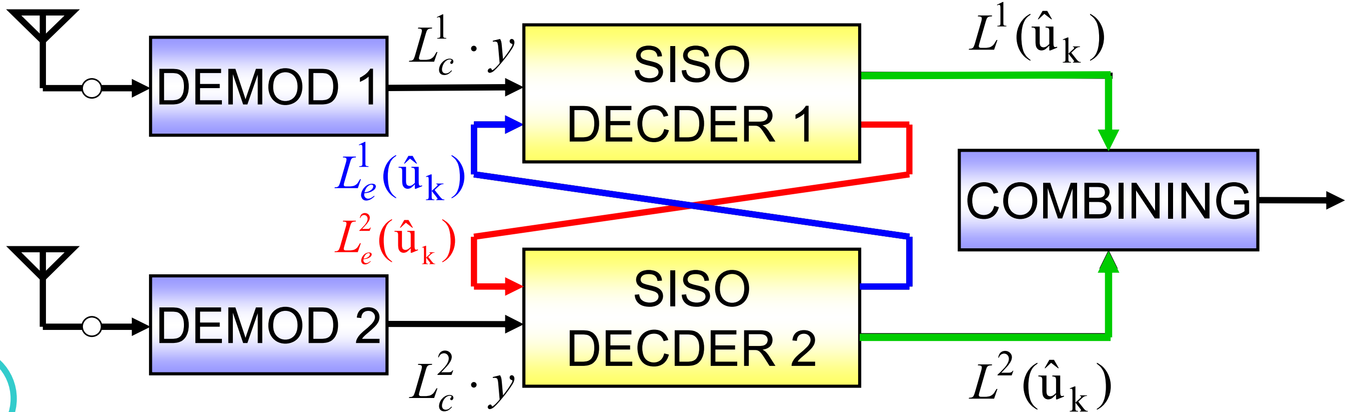
| | | |
|---|---|---|
| $L(\mathrm{u}_{\mathrm{k}})$ | : | LLR of *a priori* information |
| $L_c \cdot y$ | : | Channel state information |
| $L_e(\hat{\mathrm{u}}_{\mathrm{k}})$ | : | LLR of extrinsic information |

# Iterative decoding

- The extrinsic information obtained through decoding process on the first diversity branch is used as *a priori information* on the second branch.

$$L^1(\hat{u}_k) = \boxed{L^1(u_k)} + L_c y_1 + \boxed{L_e^1(\hat{u}_k)}$$

$$L^2(\hat{u}_k) = \boxed{L^2(u_k)} + L_c y_2 + \boxed{L_e^2(\hat{u}_k)}$$

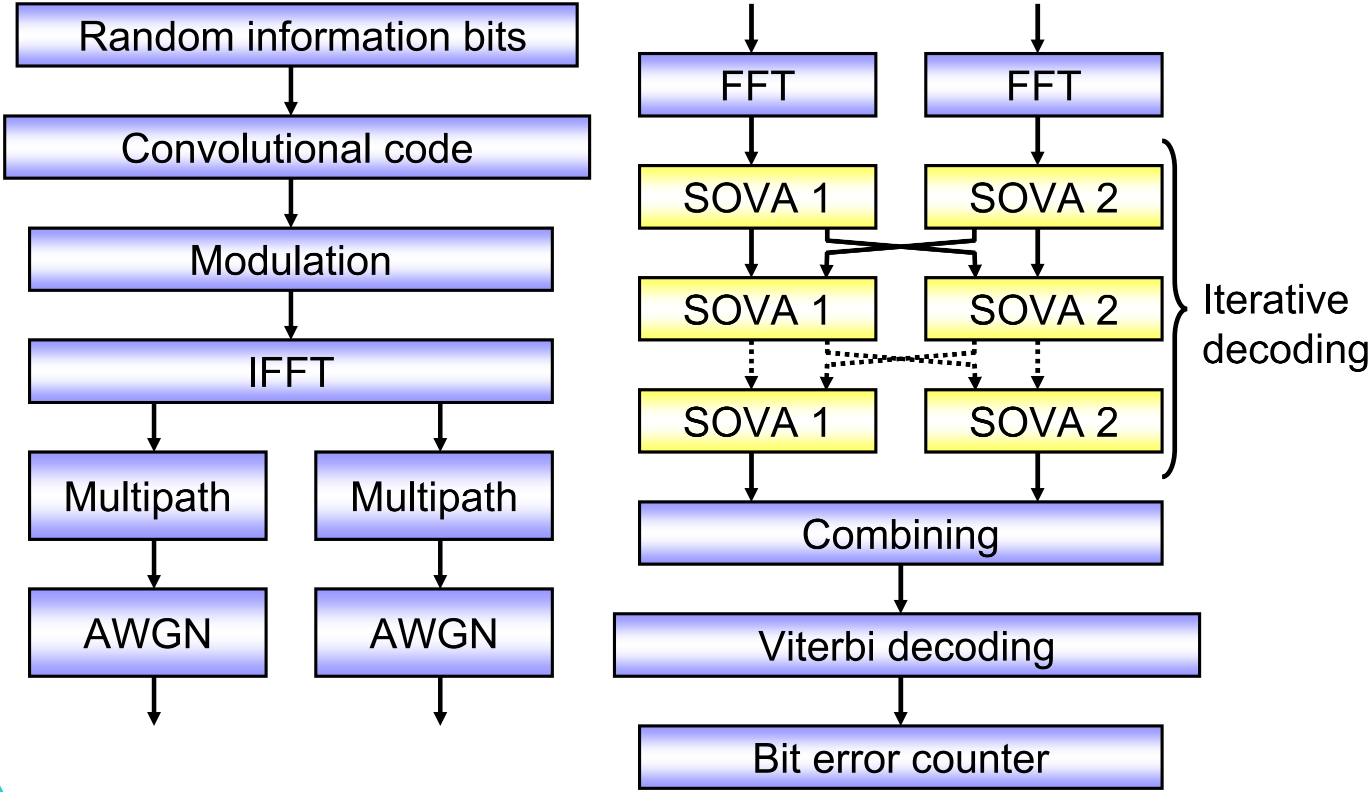


# Simulation parameters

◆ The simulation is conducted based on the parameters for the digital terrestrial TV broadcasting system in Japan.

| | |
|---|---|
| modulation | 64QAM OFDM |
| convolutional code | coding rate : 1/2<br>constraint length : 7 |
| guard interval | 31.5μs（1/32） |
| FFT size | 8192 |
| number of sub- carriers | 5617 |

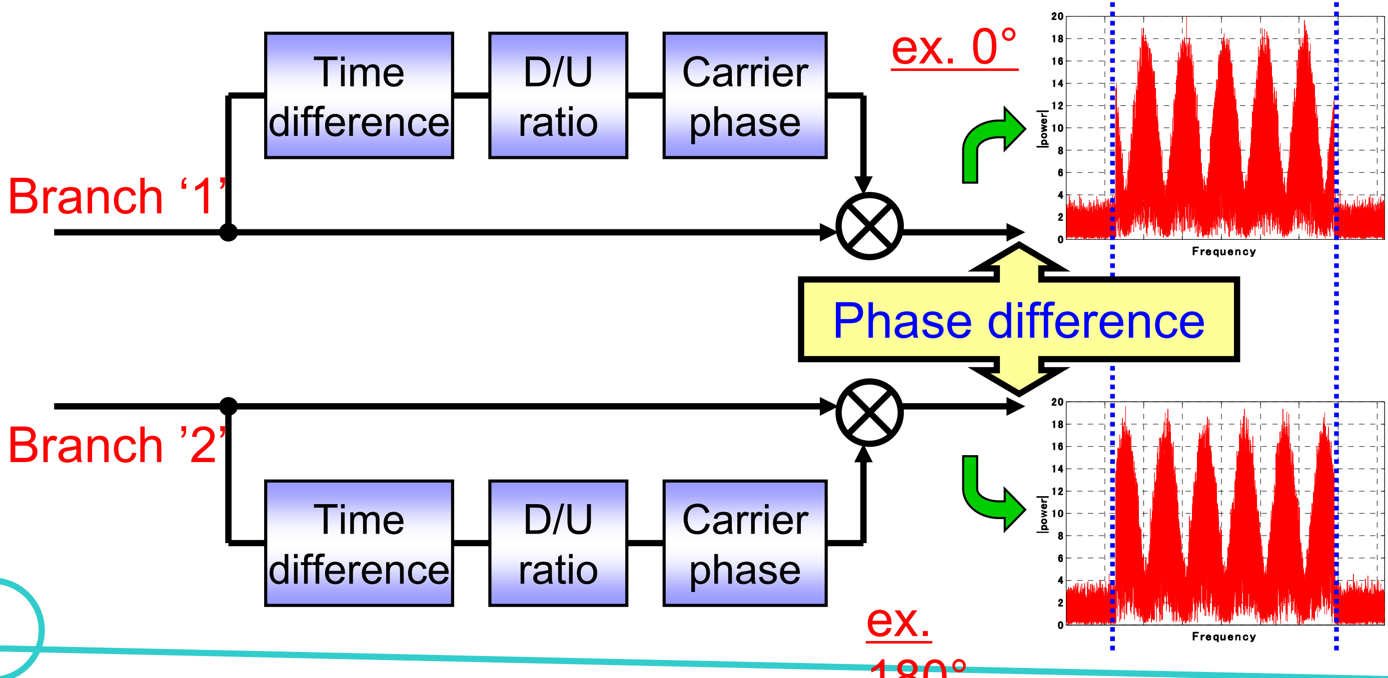
# Simulation process



# Multipath generation

◆ Two-path model as a multipath model.

◆ Time difference, D/U(Desired to Undesired signals power) ratio, and initial carrier phase are set, as parameters.



# Evaluated items(phase difference)

- The effect of phase difference is evaluated.

- Simulation of multipath

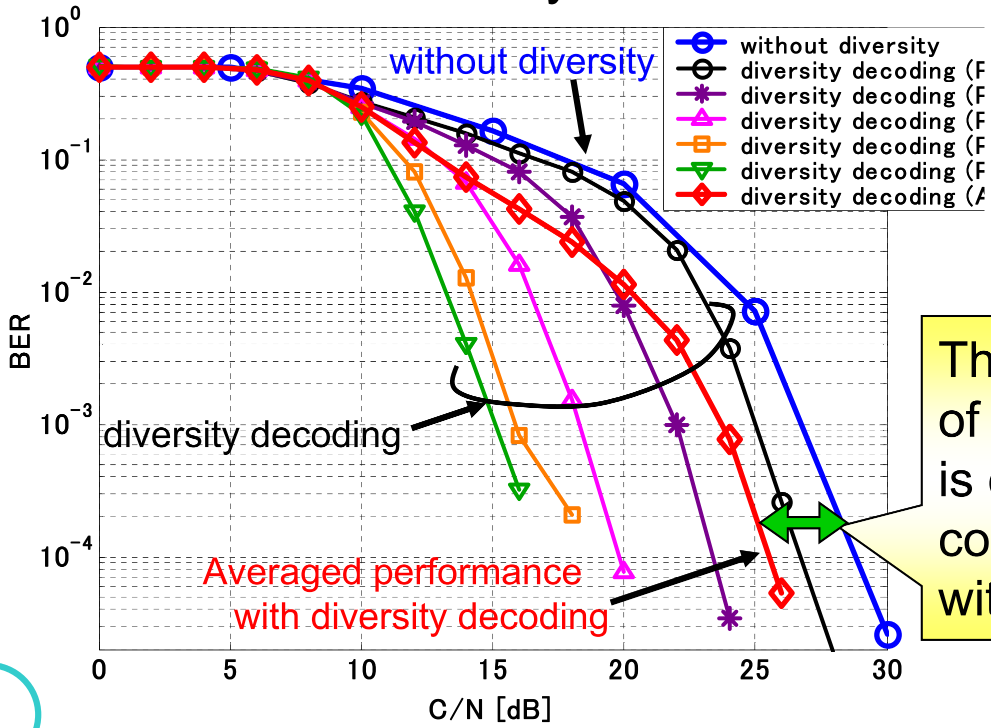
Time difference : 1[μs] , D/U : 3[dB]
Phase difference between two branches : 0 ～ 180°

※ <u>Time difference</u> and <u>D/U</u> are the same for each branch.

※ <u>Phase difference</u> is independently set in each branch.

# Simulation results

- BER performance as a parameter of the phase difference of the delayed wave between two branches.



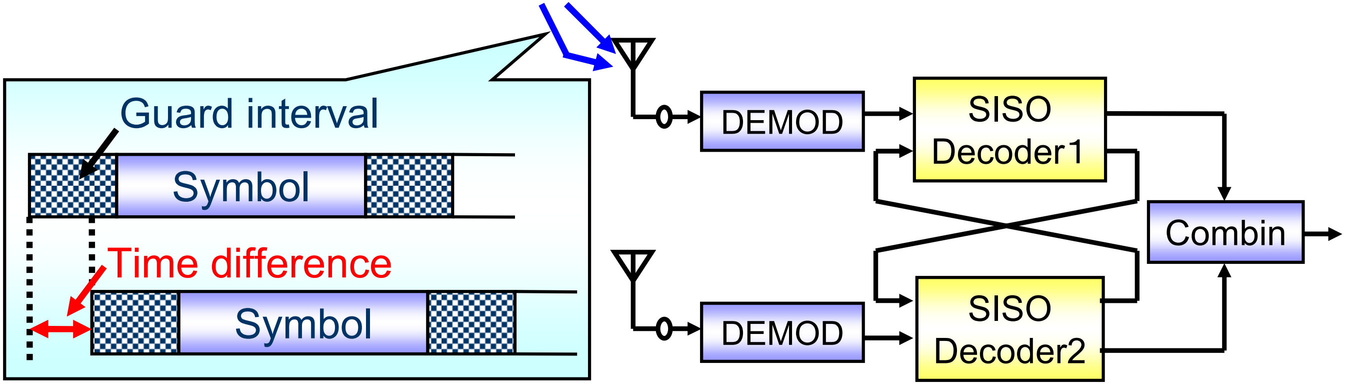

The diversity gain of about 3~4[dB] is obtained as compared with without diversity

# Evaluated items(delay time)

- The effect of time difference is evaluated.
  - Simulation of multipath

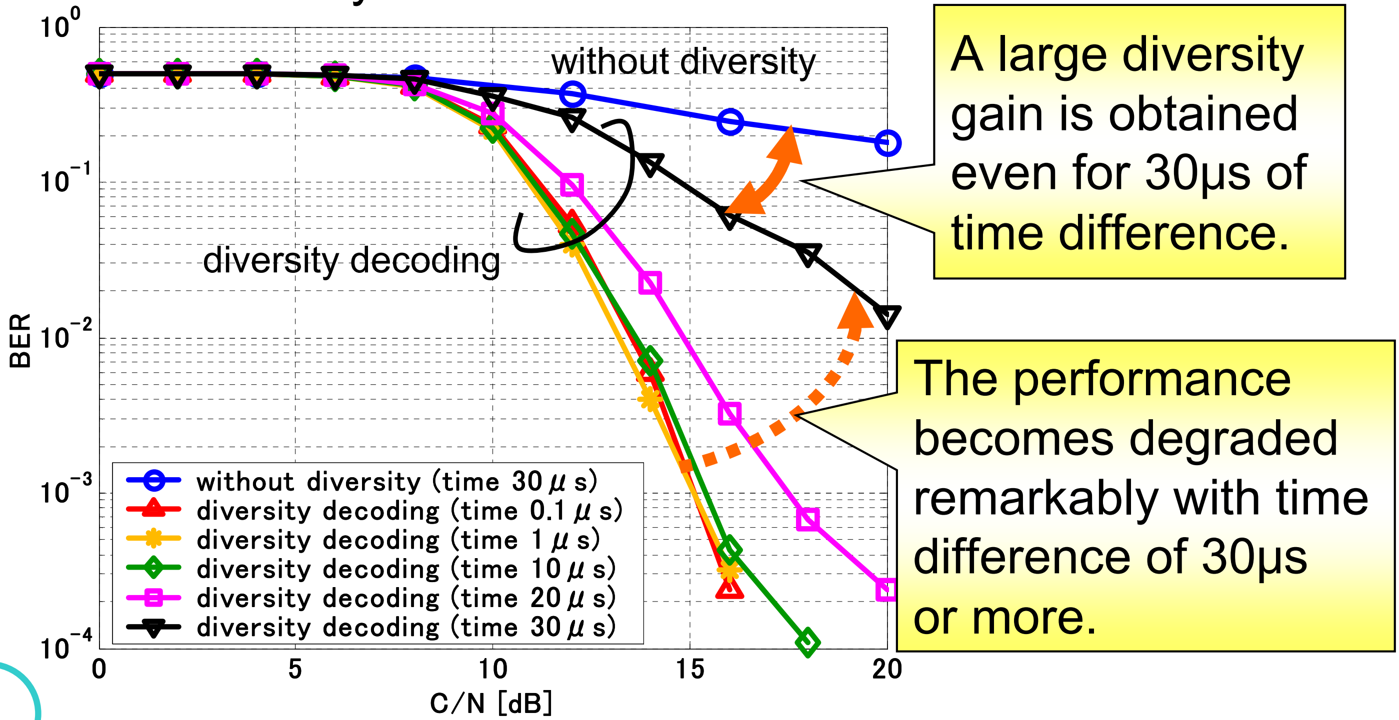
D/U : 3 [dB]
Phase difference between two branches : 180°
Time difference : 0.1 ～ 30 [μs]

※ Time difference and D/U are the same for each branch.



# Simulation results

- BER performance as a parameter of time difference of the delayed wave.

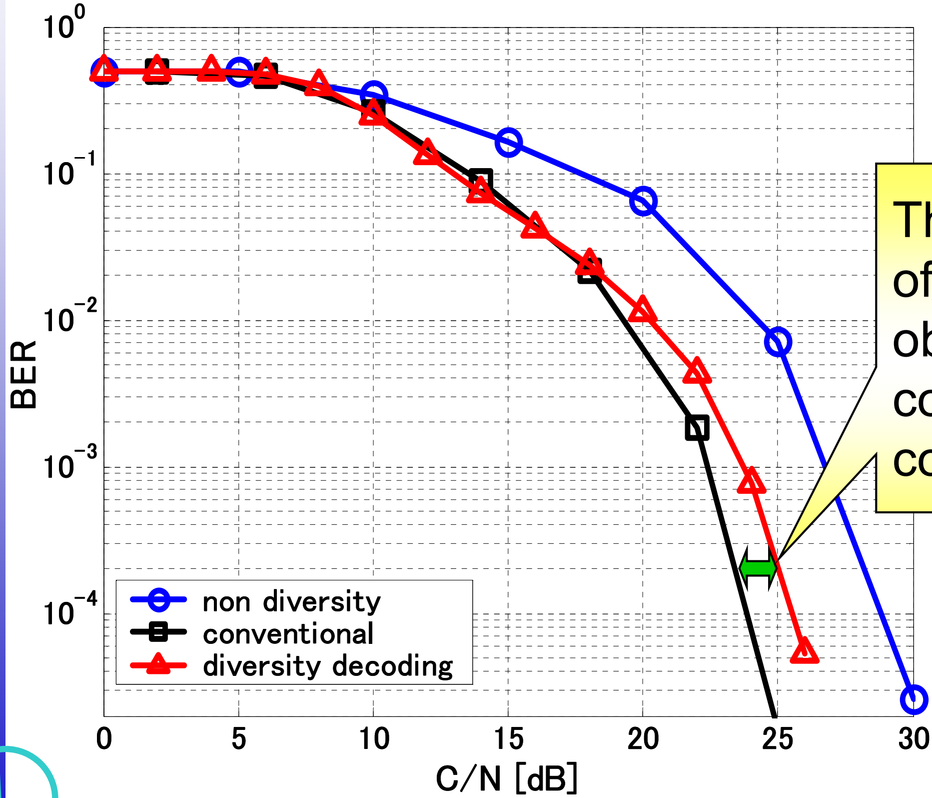


# Conclusions

- Simulation results for the bit-error-rate performance of the diversity decoding scheme under multipath environment have been presented.

- As a result, the effectiveness of diversity decoding scheme is found to be great under the multipath environment.

# Performance comparison




The low performance of about 1[dB] was obtained as compared with conventional scheme

# Pilot signal

- The intervals of pilot are every 12 sub- carriers.